Screwlike order, macroscopic chirality, and elastic distortions in high-density DNA mesophases

F. Manna,[1] V. Lorman,[1] R. Podgornik,[2,3,4] and B. Žekš[3,5]

[1]*Laboratoire de Physique Mathematique et Theorique, Universite Montpellier II, F-34095 Montpellier, France*
[2]*Department of Physics, Faculty of Mathematics and Physics, University of Ljubljana, SI-1000 Ljubljana, Slovenia*
[3]*Department of Theoretical Physics, J. Stefan Institute, SI-1000 Ljubljana, Slovenia*
[4]*LPSB/NICHHD, Building 12A, Room 2041, National Institutes of Health, Bethesda, Maryland 20892-5626, USA*
[5]*Institute of Biophysics, Medical Faculty, University of Ljubljana, SI-1000 Ljubljana, Slovenia*

We investigate a new screwlike liquid-crystalline ordering in solutions of helical biopolymers and its influence on the state of individual molecules. In the resulting mesophase translational and rotational motions of molecules are coupled in screw fluctuations. We show that in contrast to the case of conventional chiral liquid crystals the elastic distortion does not twist the screw order but leads to overwinding of individual helical molecules. This explains the peculiarities of high-density DNA mesophases.

High-density DNA mesophases are not only important from a fundamental physical point of view, as prime examples of ordering in dense solutions of long, stiff polyelectrolytes [1], but have essential repercussions also in the very basic biological processes of living matter. Various forms of stress (e.g., food deprivation) and shock (e.g., low temperatures) were shown to induce compactification of native DNA in archea, bacteria, and even eucarya, suggesting that compaction and ordering of DNA is the last-resort survival strategy [2]. Ordered phases of DNA, such as those induced by constraining and pulling of DNA fibers in aqueous solutions, have also played a crucial role in the elucidation of atomic structure of single DNA duplexes in the most important advance in (biological) sciences ever [3]. Though we do have a description and basic understanding of high-density DNA mesophases (see Ref. [1], and references therein), many important details of the nature and properties of molecular order in these phases still need to be properly sorted out. An especially interesting issue pertains to the relation between the properties of macroscopic order of high-density DNA mesophases and the nature of the microscopic symmetries of the DNA molecule, such as its chiral [4] and helical nature. It is with the latter that we will be concerned in this Rapid Communication.

Let us start by considering a (liquid-crystalline) state in which the long axes of helical molecules are correlated. This is, for example, the case in the line nematic phase of biopolymers. In this phase the rotations of individual molecules around their long axes and the translations along these axes do not brake the symmetry of the medium. The aim of this work is to show that further correlation of helical (and not simply chiral) molecules can lead to the microscopically inhomogeneous state with nm-scale period in which independent rotations and translations [Fig. 1(a) and 1(b)] are suppressed to a certain extent, but coupled rotation-translation fluctuations [Fig. 1(c)] in the form of a screwlike motion preserve the symmetry of the new ordered state. The resulting screwlike phase is induced by a single instability of the parent disordered line nematic state. In contrast to the ordering of homogeneous molecular properties (such as average dipole of a molecule, or average third-rank tensor of a tripod discotic molecule [5], or bond angle of hexatic ordering [6],

etc.), the correlation of periodic properties of a molecule (such as helical charge distribution along a DNA) does not lead to microscopically homogeneous ground state with azimuthal angular order. In addition, due to the microscopic short-wavelength helical structure of the new screwlike phase the action of a long-wavelength chiral elastic distortion does not change its macroscopic symmetry but simply renormalizes its period. To clarify the difference between (i) azimuthal angular ordering, (ii) positional ordering, and (iii) periodic helical ordering of helical molecules we introduce the properties of individual molecules which become correlated in the respective ordered phases and discuss the distribution functions dependent on the corresponding physical variables. As usual at the first step of consideration we are interested in the ground states at low temperature and weak chirality and at the second step analyze their chiral elastic distortions.

Following classical works by Klug, Crick, and Wyckoff (KCW) [7] we parametrize mass and charge distribution of a helical molecule with the length H, radius R, and pitch $p = 2\pi/k$ in the form $x = R\cos(\phi + k\zeta)$, $y = R\sin(\phi + k\zeta)$, $z = z_0 + \zeta$. Here (x, y, z) is the coordinate of a point on a helix in the laboratory frame, ζ runs along the long molecular axis ($0 \leq \zeta \leq H$), ϕ is the initial rotational phase of the helix in the
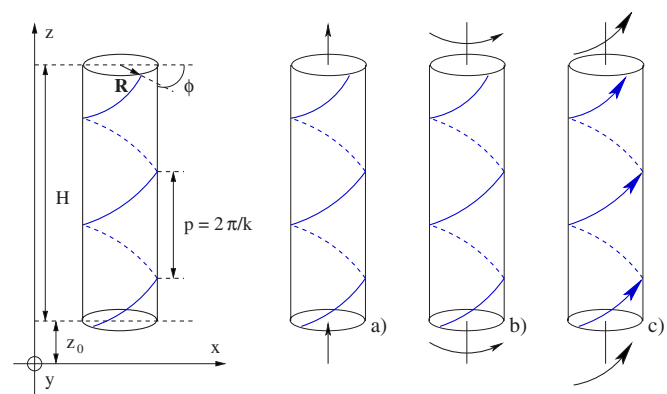

FIG. 1. (Color online) Fluctuational motion of helical molecules: (a) translational motion; (b) rotational motion; (c) coupled screwlike motion. See the text.

(x, y) plane, and z_0 is the initial translational phase of the helix with respect to the frame origin. In cylindrical coordinates (r, θ, z) the orientation of the ith helical molecule in the plane with $z = \text{const}$ is given by $\theta^i = \phi^i + k(z - z_0^i)$. The spatial relation between the ith and the jth molecules is defined by the difference $\theta^i - \theta^j = \phi^i - \phi^j - k(z_0^i - z_0^j)$. Introducing the total helical phase of the molecule $\psi^i = \phi^i - kz_0^i$, we obtain that this relation is given by the difference of total phases of the molecules $\theta^i - \theta^j = \psi^i - \psi^j$.

In a uniaxial nematic phase all three physical quantities: azimuthal angular variable ϕ, translational phase variable z_0, and total helical phase ψ, are fluctuating, but as it is evident from the definition of ψ, only two of them are linearly independent. The corresponding probability distribution function [8] $f(\psi, \phi, z_0)$, which is independent of these quantities in the disordered nematic phase, becomes dependent (e.g., Gaussian in the case of a very high order) on one of these variables in the ordered liquid-crystalline state. The dependence on the translational variable z_0 leads to the usual smectic phase [8] with the free azimuthal fluctuations of ϕ. The second possibility of ordering is the dependence of the distribution function on ϕ. It leads to the biaxial nematiclike state of helical molecules, which is in addition homogeneously polar in the (x, y) plane, translational fluctuations of z_0 being free. This state corresponds to the purely azimuthal angular ordering of molecules. It belongs to a wide class of angular states which are well-known and extensively discussed in different physical contexts, (e.g., for the bond angle of hexatic ordering [6] and for the orientation angle of a tripod discotic molecules in columnar phases [5]). The action of chirality on these microscopically homogeneous states results in a long-wavelength helical twisting of azimuthal angular variables thus changing their macroscopic symmetry. More complex structures can be generated by the coupling of angular order with additional degrees of freedom (which are out of the scope of this paper since we are interested in single instability of the disordered state). Namely, in the case of chiral columnar phases of tripod molecules, the coupling of the tripod orientational order with the crystal lattice in the plane perpendicular to columns can give rise to the "plastic" columnar phase, Moiré phase of columns, or to the twist grain boundary-like state [5]. The state induced by the azimuthal angular ordering of polar vectors was also proposed previously as a possible realization of the condensed phase of hydrated DNA fibers [9]. In the present work we are interested in the third possible liquid-crystalline phase of helical molecules which is described by the dependence of the distribution function on the total helical phase $\psi = \phi - kz_0$. Note that ψ is not an azimuthal angular degree of freedom since it depends on the translational phase and on the molecular period. The ordering of this physical quantity makes the medium periodic on the microscopic scale with the period defined by the periodicity of individual molecule. Let us show that the correlation of total helical phases of the molecules leads to a collective screwlike order in the medium. The degree of correlation depending on the anisotropy of the distribution can be characterized by the first moment [10] $P = \langle \cos \psi \rangle$, which can be taken as a module of normalized "order parameter" [8] of the phase transition from the nematic to the screw phase. In the disordered state
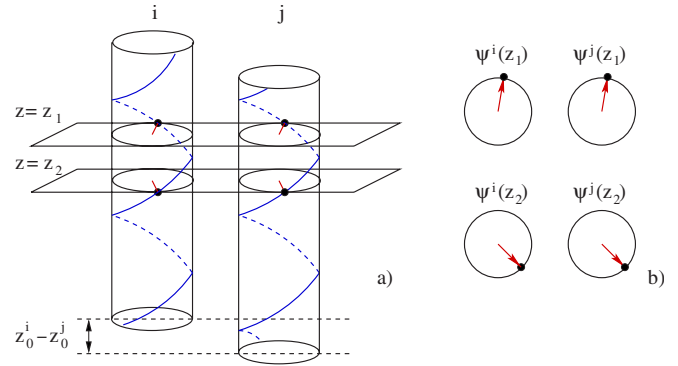

FIG. 2. (Color online) Screwlike order in the mesophase with correlated helical molecules. Though ith and jth molecules carry out screwlike fluctuational motion their total helical phases ψ^i and ψ^j are correlated in any plane $z = \text{const}$. (a) Instantaneous positions and orientations of the ith and jth molecules, $z_0^i - z_0^j$ being the difference of initial translational phases. Correlation in two different sections $z = z_1$ and $z = z_2$ is shown. (b) $\mathbf{R}^i$ and $\mathbf{R}^j$ vectors remain parallel in the planes $z = z_1$ and $z = z_2$ though their common direction varies during a screwlike motion.

P vanishes, in the ordered state with screwlike symmetry, for a Gaussian-like distribution of total helical phases of molecules, $0 < P < 1$, and $P = 1$ for perfectly correlated helices (i.e., for a δ function-like distribution).

In contrast to the nematic state, to the smectic state and to the state with angular order the molecules in the screw phase cannot freely shift along the long molecular axis z, nor freely rotate around this axis in the (x, y) plane, but they can freely perform screwlike motion. Coupled rotation-translation fluctuation does not break the correlation between the helices [Fig. 2(a)].

The correlation can also be illustrated also by using the geometrical interpretation of the KCW description of helical molecules. Let us introduce $\mathbf{R}^i = (x^i, y^i)$, the transverse polar vector which runs from the molecule axis to the point of intersection of the helix with the plane $z = \text{const}$. In the perfectly correlated state, transverse polar vectors of the molecules are parallel $\mathbf{R}^i \| \mathbf{R}^j$ in any plane $z = \text{const}$, but the common direction of these vectors turns around the z axis from one plane to another [Fig. 2(b)]. Macroscopic symmetry of the screw phase is described by the space group $\infty_1 22$ (in the international crystallographic notation). The z axis of the medium of correlated helical molecules becomes a continuous screw axis with the periodicity $T = 2\pi/K$ equal to the pitch $p = 2\pi/k$ of individual helical molecules: translation with any vector d along the axis followed by rotation with the angle $-Kd$ around the axis does not change the structure of the screw phase. Note that though the screw phase is periodic, its continuous screw axis forbids any scalar density modulation along the z direction. Consequently, this type of ordering does not lead to smectic layers formation. For further, more detailed discussion of the physical properties of the screw mesophase, we perform a standard group theory analysis of the phase transition from the nematic state to this phase. The irreducible representation of the symmetry group of the disordered state responsible for the considered transition is two dimensional and is spanned by the following basic functions:

$$\boldsymbol{\xi} = (\hat{\mathbf{e}}_x - i\hat{\mathbf{e}}_y)\exp(iKz); \quad \boldsymbol{\xi}^* = (\hat{\mathbf{e}}_x + i\hat{\mathbf{e}}_y)\exp(-iKz), \quad (1)$$

where $\hat{\mathbf{e}}_x$ and $\hat{\mathbf{e}}_y$ stand for the unit vectors along the x and y axes, respectively. Order parameter components corresponding to basic functions (1) are

$$\eta = P\exp(i\Phi); \quad \eta^* = P\exp(-i\Phi). \quad (2)$$

Thus, the transition to the screw phase is driven by the formation of a macroscopic transverse polar vector wave $\mathbf{P}(z)$ with the circular polarization and the wave vector $\mathbf{K}$ directed along the common axis of helical molecules:

$$\mathbf{P}(z) = \eta\boldsymbol{\xi} + \eta^*\boldsymbol{\xi}^* = P\cos(Kz + \Phi)\hat{\mathbf{e}}_x + P\sin(Kz + \Phi)\hat{\mathbf{e}}_y. \quad (3)$$

Here $P = \langle\cos\psi\rangle$ is the order parameter module defined above and Φ is the Goldstone phase of the order parameter [11].

One of the peculiarities of the screw phase is its apparent similarity to other liquid-crystalline phases (especially to the nematic one) in a number of experiments. Indeed, classical x-ray diffraction cannot evidence the periodicity of the screw phase because the density modulation in this state is forbidden by the continuous screwlike symmetry axis. Helical periodicity of the screw phase is equivalent to the polar vector wave [Eq. (3)] and thus can give a nonzero contribution only to the resonant x-ray diffraction near the absorption gap of one of the atoms constituting helical biomolecule. Resonant diffraction which uses different polarizations of synchrotron radiation [12,13] had permitted to evidence the multilayer tilt periodic structures in ferrielectric liquid crystals [14]. It can be useful for the explicit determination of the screw-phase structure in the condensed DNA solutions. However, if the helical period of the molecules and consequently of the ordered screw phase is much longer and is comparable with the visible light wavelength, its periodicity can be visualized in a simple way. For this aim one can use optical polarizing microscopy which is sensitive to the orientations of molecular fragments. The value of molecular pitch of DNA, actin, or helical viruses does not permit one to perform this experiment, but several flagellar filaments have helical shape with the well-controlled periodicity exactly in the μm range. A recent excellent polarizing microscopy experiment by Barry *et al.* has shown the well-defined striped birefringent pattern in the condensed suspensions of flagella isolated from prokaryotic bacteria [15]. The pattern periodicity is equal to the helical pitch of individual flagella. Thus, the pattern is associated with the one-dimensional variations of the local optical axis direction in the phase formed by the correlated helical filaments. In addition, differential interference contrast microscopy of the flagella suspensions confirmed that there is no density modulation associated with the observed order, in perfect accord with the properties of the screw-phase model. And, finally, dynamics study of a fluorescently labeled flagellum indicated that the translational diffusion in the suspension is coupled to rotational diffusion in a screwlike motion in agreement with the type of symmetry-preserving motion [Fig. 1(c)] expected in the screw phase.

In what follows we will describe another striking feature of the screw phase, namely the overwinding of individual

helical molecules under the action of the large-scale elastic distortion due to the macroscopic chirality of the medium. This result is quite different with respect to the action of chirality on the conventional locally homogeneous liquid-crystal order. It is due to two factors: (i) the order in the screw phase is already wavelike even at the microscopic scale; (ii) most of helical biomolecules and supermolecular assemblies are soft enough to adjust their pitches under the action of the elastic torque. To illustrate this property we study a minimal thermodynamic model of the phase transition to the screw phase. The free energy of the transition invariant with respect to the symmetry operations of the parent line nematic phase is given by

$$\mathcal{F} = \frac{1}{L}\int\left[\frac{a}{2}\eta\eta^* + \frac{b}{4}(\eta\eta^*)^2 - \lambda\left(\eta\frac{\partial\eta^*}{\partial z} - \eta^*\frac{\partial\eta}{\partial z}\right)\right.$$
$$\left. + \frac{g}{2}\frac{\partial\eta}{\partial z}\frac{\partial\eta^*}{\partial z}\right]dz$$
$$= \frac{1}{L}\int\left[\frac{a}{2}P^2 + \frac{b}{4}P^4 - \lambda P^2\frac{\partial\Phi}{\partial z} + \frac{g}{2}P^2\left(\frac{\partial\Phi}{\partial z}\right)^2\right]dz. \quad (4)$$

Here L is the sample dimension along the z axis. Free energy (4) contains a nonzero Lifshitz term due to the macroscopic chirality of the medium of helical biomolecules [16]. The equations of state of the minimal model have two possible solutions: (a) parent disordered phase for $P = 0$; and (b) screw phase for $P \neq 0$. Far in the ordered screw phase the module P of the order parameter can be considered constant and the inhomogeneous part of free energy (4) describes spontaneous elastic distortion of the local structure:

$$\mathcal{F}_{dist} \simeq \frac{1}{L}\int\left[-\tilde{\lambda}\frac{\partial\Phi}{\partial z} + \frac{\tilde{g}}{2}\left(\frac{\partial\Phi}{\partial z}\right)^2\right]dz, \quad (5)$$

where $\tilde{\lambda} = \lambda P^2$ and $\tilde{g} = gP^2 > 0$. Standard Euler-Lagrange procedure gives the spatial dependence for the phase Φ of the order parameter in the form $\Phi = qz + \Phi_0$, with modulation wave vector $q = \tilde{\lambda}/\tilde{g}$. In striking contrast with the case of locally homogeneous order (i.e., nematic or hexatic ones) this dependence does not lead to the twisting of the screwlike order of correlated helical molecules. Indeed, macroscopic transverse polar vector wave $\mathbf{P}(z)$ [Eq. (3)], which expresses the screw phase structure, becomes

$$\mathbf{P}(z) = P\cos[(K + q)z + \Phi_0]\hat{\mathbf{e}}_x + P\sin[(K + q)z + \Phi_0]\hat{\mathbf{e}}_y. \quad (6)$$

It means that the wave vector of the screwlike order is simply renormalized by the influence of macroscopic chirality without other qualitative changes of the ordered structure. For positive $\tilde{\lambda}$ it leads to overwinding, i.e., to the decrease of the screw structure period. No twisting of the structure occurs in the mesophase with correlated helical molecules. Taking into account the fact that chiral terms in the free energy are usually small [8], the screw phase pitch variation from $T = 2\pi/K$ to $T' = 2\pi/(K + q)$ should also be small. The value of q in chiral liquid crystals usually depends on the molecular chirality, but also depends on thermodynamic param-

eters (temperature for cholesteric or SmC^* liquid crystals, density for lyotropic mixtures, and colloidal solutions). In the condensed solutions of DNA fragments [17] q is a growing function of the DNA density. If the fragments are correlated in the screw phase the structure periodicity decreases with increasing density. Note that the most probable scenario of the screw structure period variation is the correlated overwinding of individual helical molecules from $p = 2\pi/k$ to $p' = 2\pi/(k+q)$. This overwinding with the density increase, resulting in continuous decrease of the number of nucleotides per helix turn, was observed in the aqueous solutions of 50 nm long DNA fragments [18]. In the same density range condensed DNA solutions show no twisting of the ordered structure [19]. Both remarkable properties observed in the DNA solutions constitute strong indication of the screw-like order formation in this system.

To show that the DNA molecules are deformable enough to adjust their pitches in the screw phase we estimate the elastic energy transmitted to an individual molecule and the elastic torque exerted on the DNA fragment by the chiral medium. The elastic properties of DNA have been studied extensively in a series of single molecule experiments with optical tweezers (see, for example, [20] and references therein). In the simplest case the model is an inextensible wormlike chain with the two terms expressing the energies of bending and overwinding, respectively [21],

$$\frac{E}{k_B T} = \int_0^{l_0} \left[\frac{A}{2} \left(\frac{dt}{ds} \right)^2 + \frac{C}{2} \Omega^2 \right] ds. \qquad (7)$$

Here $A \simeq 50$ nm is the bending persistence length under tensions less then 10 pN$= 10^{-11}$ N and in ≥ 10 mM univalent salt solutions. Twisting (under- or overwinding) is described by the rotation angle per arc length of base pairs around $\hat{\mathbf{t}}$ (tangent vector of the chain); an unperturbed DNA has repeat unit $p \simeq 3.4$ nm and thus a twist rate (wave vector of the molecular helix) $k \simeq 1.84$ nm^{-1}. Deviation Ω of the twisting away from this rate is defined as the superwinding wave vector q added to the k. The length of the DNA fragments used in the x-ray experiment [18] was close to the DNA bending persistence length, thus no additional bending has been found [18]. Constant $C \simeq 75$ nm is the persistence length for twist fluctuations [20,21]. In the region of stability of the fluid mesophases, the DNA repeat unit variation is of the order of few percent, $q \simeq 0.1$ nm^{-1}, then the overwinding energy per length is $E_{ow}/l_0 \simeq 0.375 k_B T$ nm^{-1}. We also estimate the elastic torque $M \simeq E_{ow}/l_0 \Omega \simeq 3.75 k_B T$ exerted on the molecule. At $T = 300$ K the relation $1 k_B T = 4.1$ pN nm gives $M \simeq 15$ pN nm. Both energy and torque estimated are available in the system of DNA molecules with nm-scale pitches and typical forces of several pN which do not change their nature or biochemical properties [20].

---

[1] R. Podgornik, H. H. Strey, and V. A. Parsegian, Curr. Opin. Colloid Interface Sci. **3** 534 (1998).

[2] A. Minsky, E. Shimoni, and D. Frenkiel-Krispin, Nat. Rev. Mol. Cell Biol. **3**, 50 (2002).

[3] R. E. Franklin and R. G. Gosling, Nature (London) **171**, 740 (1953).

[4] R. D. Kamien and A. J. Levine, Phys. Rev. Lett. **84**, 3109 (2000).

[5] Gu Yan and T. C. Lubensky J. Phys. II **7**, 1023 (1997).

[6] R. D. Kamien, J. Phys. II **6**, 461 (1996).

[7] A. Klug, F. H. C. Crick, and H. W. Wyckoff, Acta Crystallogr. **11**, 199 (1958); W. Cochran, F. H. C. Crick, and V. Vand, *ibid.* **5**, 581 (1952).

[8] P. G. de Gennes and J. Prost, *The Physics of Liquid Crystals* (Oxford University Press, New York, 1993).

[9] H. M. Harreis, A. A. Kornyshev, C. N. Likos, H. Lowen, and G. Sutmann, Phys. Rev. Lett. **89**, 018303 (2002); V. Lorman, R. Podgornik, and B. Žekš, *ibid.* **87**, 218101 (2001); A. A. Kornyshev, D. J. Lee, S. Leikin, A. Wynveen, and S. B. Zimmerman, *ibid.* **95**, 148102 (2005).

[10] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, London, 1958).

[11] P. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, New York, 1995).

[12] V. E. Dmitrienko, Acta Crystallogr., Sect. A: Found. Crystallogr. **39**, 29 (1983).

[13] A. M. Levelut and B. Pansu, Phys. Rev. E **60**, 6803 (1999).

[14] L. S. Hirst *et al.*, Phys. Rev. E **65**, 041705 (2002).

[15] E. Barry *et al.*, Phys. Rev. Lett. **96**, 018305 (2006).

[16] One should clearly distinguish chirality of helical molecules and their helicity. Chirality is related to the nonzero value of the pseudoscalar property averaged over the molecule and does not imply any periodicity. Helicity is related to the periodic helical mass and charge distribution of the KCW type. All helical molecules are chiral, but there exist chiral molecules which are not helical.

[17] F. Livolant and A. Leforestier, Prog. Polym. Sci. **21**, 1115 (1996).

[18] D. Durand, J. Doucet, and F. Livolant, J. Phys. II **2**, 1769 (1992).

[19] H. H. Strey *et al.*, Phys. Rev. Lett. **84**, 3105 (2000).

[20] R. Lavery *et al.*, J. Phys.: Condens. Matter **14**, R383 (2002).

[21] J. F. Marko, Europhys. Lett. **38**, 183 (1997).